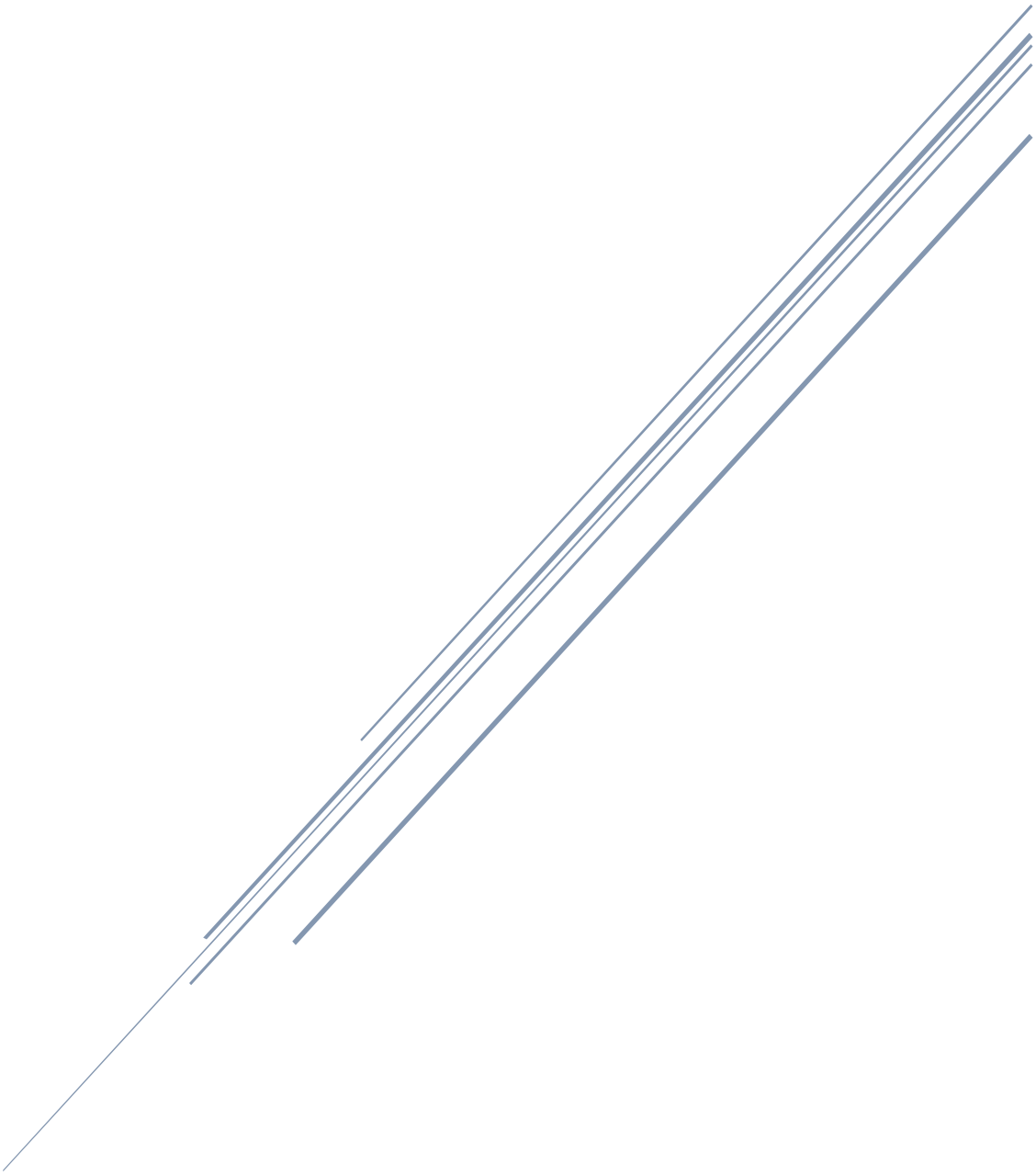
FRACTIONS

GCSE Fractions



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GCSE Mathematics: Fractions

Success Criteria for Multiplication of Numerical Fractions

Step	Instruction	Example
1a	Examine the numerators and denominators to see if you can cancel anything down. You must cancel a denominator with a numerator. You are not allowed to cancel denominator with denominator or numerator with numerator.	$\frac{7}{8} \times \frac{20}{21} =$
1b	Note here that we can use any denominator to cancel down any numerator. Seven goes into both the numerator and the denominator, in this case, of opposite fractions.	$\frac{7}{8} \times \frac{20}{21} =$ 7 goes into 7 one time. 7 goes into 21 three times.
1c	It could be that the numerator and the denominator of the same fraction cancels down, but again in this case, it is the opposite fractions that cancel. That is fine.	$\frac{1}{8} \times \frac{20}{3} =$ 4 goes into 8 two times. 4 goes into 20 five times.
1d	Here we have a far easier calculation to do when we are multiplying. If you notice, as well, there are no other ways the fraction can cancel down, so once we have multiplied the numerators together and the denominators together, we have found the simplest possible solution. Just to be on the safe side, you should check if there are other ways that you can cancel down at this stage.	$\frac{1}{2} \times \frac{5}{3} =$
2	Multiply top × top, bottom × bottom. There is no need to cancel down as you have already done this.	1 times 5 is 5 $\frac{1}{2} \times \frac{5}{3} = \frac{5}{6}$ 2 times 3 is 6

Multiplication of numerical fractions

Now try these using the cancelling down and then multiplying method.

1 $\frac{24}{36} \times \frac{9}{16} =$

2 $\frac{35}{40} \times \frac{16}{28} =$

3 $\frac{48}{60} \times \frac{45}{72} =$

4 $\frac{28}{35} \times \frac{15}{16} =$

5 $\frac{42}{49} \times \frac{63}{81} =$

6 $\frac{25}{65} \times \frac{26}{30} =$

7 $\frac{18}{45} \times \frac{15}{81} =$

8 $\frac{56}{72} \times \frac{18}{35} =$

9 $\frac{18}{25} \times \frac{55}{99} =$

10 $\frac{36}{60} \times \frac{48}{50} \times \frac{110}{121} =$

11 $\frac{35}{36} \times \frac{42}{45} \times \frac{54}{98} =$

12 $\frac{32}{68} \times \frac{51}{81} \times \frac{9}{16} =$

13 $\frac{27}{64} \times \frac{44}{99} \times \frac{16}{30} =$

14 $\frac{14}{36} \times \frac{22}{32} \times \frac{66}{121} \times \frac{24}{35} =$

15 $\frac{64}{81} \times \frac{9}{16} \times \frac{18}{24} \times \frac{22}{33} =$

16 $\frac{45}{51} \times \frac{9}{48} \times \frac{60}{63} \times \frac{3}{8} \times \frac{34}{75} =$

Algebraic Question as a Challenge

Use exactly the same rules here:

C1 $\frac{6a^3b^2}{a^9} \times \frac{7a^5c^7}{12c^3d^5} \times \frac{a^2d^6}{e^4f^3} \times \frac{9b^8d^4}{bf^2} \times \frac{ae^3f^6}{15c^4} \times \frac{10bc^5}{14f} =$

Success Criteria for Division of Fractions

Stage	Instruction	Example
1	Look at the question	$\frac{12}{39} \div \frac{15}{26} =$
2	Remember that dividing by any number is the same as multiplying it by its reciprocal so in this case, 15 over 26 becomes 26 over 15. ONLY and ALWAYS change the last fraction… otherwise it doesn't work. We sometimes use **K**eep **C**hange **F**lip or **KCF** to help us to remember what is going on here.	$\frac{12}{39} \div \frac{15}{26} = \frac{12}{39} \times \frac{26}{15}$
3	Cancel down any numbers that will allow. I have demonstrated three into 12 and 3 into 15 here. Obviously, 26 and 39 are both in the 13 times table so consequently cancel down to 2 and 3.	3 goes into 12 four times. $\frac{12}{39} \times \frac{26}{15} = \frac{4}{3} \times \frac{2}{5}$ 3 goes into 15 five times.
4	Multiply top × top, bottom × bottom	$\frac{4}{3} \times \frac{2}{5} = \frac{4 \times 2}{3 \times 5} = \frac{8}{15}$

Division of Numerical Fractions

1 $\frac{3}{5} \div \frac{9}{16} =$

2 $\frac{8}{9} \div \frac{7}{12} =$

3 $\frac{15}{21} \div \frac{10}{14} =$

4 $\frac{24}{35} \div \frac{16}{21} =$

5 $\frac{8}{11} \div \frac{7}{12} =$

6 $\frac{15}{17} \div \frac{10}{34} =$

7 $\frac{9}{13} \div \frac{18}{39} =$

8 $\frac{12}{35} \div \frac{15}{49} =$

9 $\frac{8}{13} \div \frac{40}{55} =$

10 $\frac{34}{121} \div \frac{51}{77} =$

11 $\frac{18}{25} \div \frac{54}{65} =$

12 $\left(\frac{68}{75} \times \frac{15}{34}\right) \div \frac{8}{15} =$

13 $\left(\frac{16}{27} \times \frac{9}{24}\right) \div \frac{8}{15} =$

14 $\left(\frac{7}{8} \times \frac{10}{21}\right) \div \frac{15}{36} =$

15 $\left(\frac{18}{25} \div \frac{12}{35}\right) \div \left(\frac{18}{35} \div \frac{12}{25}\right) =$

16 $\left(\frac{48}{51} \div \frac{32}{34}\right) \div \left(\frac{15}{16} \div \frac{45}{64}\right) =$

Challenge Question

$$\left(\left(\frac{3}{4}\right)^2 \times \left(\frac{5}{6}\right)^2\right) \div \left(\frac{3}{5} \div \frac{4}{5}\right) =$$

Success Criteria for Converting Mixed Numbers to Improper Fractions

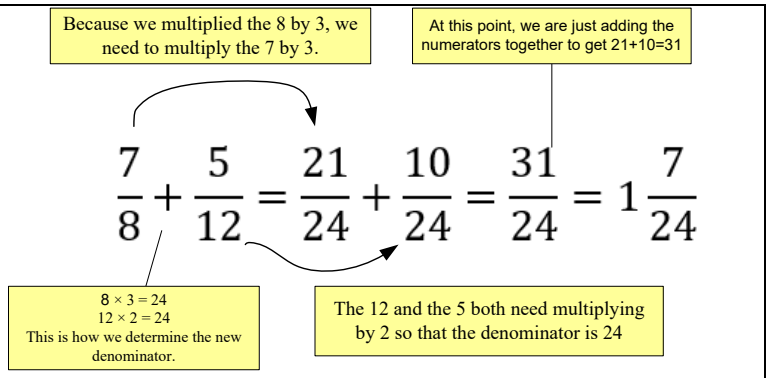
Step	Instruction	Example
1	Find the number of fractions in the whole number by multiplying the denominator by the whole number.	$4\frac{3}{5}$ $4 \times 5 = 20$
2	Add the numerator to the number you have found for step 1.	$4\frac{3}{5}$ $(4 \times 5) + 3 = 20 + 3$ $= 23$
3	Write the answer to step 2 as the numerator and keep the denominator the same.	$4\frac{3}{5} = \frac{23}{5}$
Note	If you are good with algebra, you might want to remember this instead: $a\frac{b}{c} = \frac{(a \times c)+b}{c}$	$4\frac{3}{5} = \frac{(4\times5)+3}{5} = \frac{23}{5}$

Convert these mixed numbers into improper fractions

1	$6\frac{2}{5} =$	7	$6\frac{2}{5} =$
2	$6\frac{2}{5} =$	8	$6\frac{2}{5} =$
3	$6\frac{2}{5} =$	9	$6\frac{2}{5} =$
4	$6\frac{2}{5} =$	10	$6\frac{2}{5} =$
5	$6\frac{2}{5} =$	11	$6\frac{2}{5} =$
6	$6\frac{2}{5} =$	12	$6\frac{2}{5} =$

Success Criteria for Addition and Subtraction of Fractions

Step	Instruction	Example
1	Check what type of calculation you are looking to do: a Same denominator b Related denominator c Unrelated denominator	
a	Same denominator – Keep the denominator and add up or take away the numerators. Here you can think of each fraction as if it was an object and you are basically adding or taking away those objects. 3 bricks add 9 bricks is 17 bricks. 11 bricks take 5 bricks is 6 bricks.	$\frac{3}{17}+\frac{9}{17}=\frac{3+9}{17}=\frac{12}{17}$ $\frac{11}{17}-\frac{5}{17}=\frac{11-5}{17}=\frac{6}{17}$
b	Related denominators are where one denominator is a multiple of the other. In this example, 14 is a multiple of 7 as 2×7=14. The aim is to change both denominators to being the same.	Multiply the numerator by 2 6×2=12 $\frac{6}{7}-\frac{5}{14}=\frac{12}{14}-\frac{5}{14}=\frac{7}{14}=\frac{1}{2}$ At this point, you have two fractions with the same denominator. Multiply the denominator by 2 7×2=14
c	Unrelated denominators are ones where the denominators are not multiples or factors of each other. In this case, you need to convert both fractions so that they have the same denominator. Sometimes, this will involve multiplying by	Because we multiplied the 8 by 12, we need to multiply the 7 by 12. At this point, we are just adding the numerators together to get 84+40=124 $\frac{7}{8}+\frac{5}{12}=\frac{84}{96}+\frac{40}{96}=\frac{124}{96}=1\frac{28}{96}=1\frac{7}{24}$ 8 × 12 = 96 12 × 8 = 96 This is how we determine the new denominator. The 12 and the 5 both need multiplying by 8 so that the denominator is 96

	the opposite denominator as in the example shown opposite. In this particular case though, it would have been better to find the lowest common multiple and multiply by what is needed to achieve that.	

Addition and Subtraction of Fractions

1 $\frac{3}{8}+\frac{7}{8}=$

2 $\frac{16}{19}-\frac{7}{19}=$

3 $\frac{3}{6}+\frac{7}{12}=$

4 $\frac{1}{8}+\frac{5}{24}=$

5 $\frac{7}{12}+\frac{9}{30}=$

6 $\frac{7}{60}+\frac{13}{15}=$

7 $\frac{9}{10}-\frac{8}{15}=$

8 $\frac{11}{12}+\frac{3}{8}=$

9 $\frac{5}{18}+\frac{5}{6}=$

10 $\frac{3}{10}+\frac{7}{10}=$

11 $\frac{11}{15}-\frac{18}{75}=$

12 $\frac{6}{17}+\frac{2}{3}=$

13 $\frac{4}{9}+\frac{5}{6}=$

14 $\frac{11}{12}-\frac{59}{60}=$

15 $\frac{4}{5}+\frac{4}{7}=$

16 $\frac{8}{15}+\frac{2}{5}=$

17 $\frac{3}{4}+\frac{5}{6}=$

18 $\frac{7}{8}-\frac{5}{32}=$

19 $\frac{6}{7}+\frac{11}{42}=$

20 $\frac{4}{7}+\frac{14}{15}=$

More Challenging Questions

C1 $\frac{2}{5}+\frac{3}{6}-\frac{4}{7}+\frac{5}{8}=$

C2 $3-\frac{2}{5}\left(\frac{6}{7}\div\frac{9}{25}\right)=$

C3 $4\frac{3}{4}+\left(2\frac{2}{5}\right)^3-\left(3\frac{5}{8}\div 1\frac{3}{4}\right)^2=$

C4 $1\frac{2}{3}+4\frac{3}{5}\times 6\frac{2}{3}-2\frac{1}{2}\div 1\frac{1}{5}^2$

Terminating Fractions

A terminating fraction is one which can be written as a decimal eg $\frac{7}{20}$ is a terminating fraction as its decimal equivalent is 0.35. You can tell if a fraction is a terminating fraction if it only has prime factors of 2 or 5.

Success Criteria for Converting a Terminating Fraction into a Decimal.

Step	Instruction	Example
1	Divide the numerator by the denominator	$\frac{37}{40} = 0.925$

Success Criteria for Converting a Decimal into a Fraction

Step	Instruction	Example
1	Count the number of digits after the decimal point.	0.50382 is 5 digits behind the decimal point
2	Write the digits behind the decimal point as the numerator	50382
3	Write 10 to the power of the number of digits behind the decimal point as the denominator	$\frac{50382}{10^5} = \frac{50382}{100000}$
4	Cancel down the fraction by dividing both top and bottom by their greatest common divisor or highest common factor (these mean the same thing)	$\frac{25191}{50000}$

Convert these into decimals

1 $\frac{7}{10} =$

2 $\frac{28}{100} =$

3 $\frac{107}{500} =$

4 $\frac{127}{1000} =$

5 $\frac{16}{40} =$

6 $\frac{37}{80} =$

7 $\frac{162}{400} =$

8 $\frac{87}{500} =$

9 $\frac{37}{50} =$

10 $\frac{849}{10000} =$

Convert these into fractions

1 0.7

2 0.62

3 0.731

4 0.5564

5 0.51

6 0.352

Recurring Decimals to Fractions

A recurring decimal is rational number in that it can be written as a fraction. Sometimes, recurring decimals are numbers like $0.\dot{3}$ or $0.\dot{2}487\dot{9}$ which are easy to sort out because you just need to figure out the fraction equivalent to the recurrence. On other occasions though, it slightly more complicated for fractions of the form $0.23\dot{1}759\dot{3}$ as you need to separate the 0.23 from the recurrence, do the calculations and then put them back together.

Success Criteria for Converting Recurring Decimals to Fractions (the Easy Type)

Step	Instruction	Example
1	Label the recurring part of your fraction with a letter. Here, I have chosen b. You need to understand that $0.\dot{6}1\dot{2}$ means: $0.621621621612\dot{6}1\dot{2}$ repeating forever.	Convert $0.\dot{6}1\dot{2}$ into a fraction. Let $b = 0.\dot{6}1\dot{2}$
2	Count the number of digits in the recurring string of numbers.	First digit is 6, second digit is 1, third digit is 2 so there are 3 digits in the string.
3	Multiply b by 10^n where n is the number that you found in step 2	$1000b = 612.\dot{6}1\dot{2}$
4	Subtract b from both sides to leave just an integer on the right	$1000b - b = 612.\dot{6}1\dot{2} - 0.\dot{6}1\dot{2}$ $999b = 612$
5	Rearrange the equation to get b= a fraction	$b = \frac{612}{999}$
6	Simplify the fraction by cancelling it down	$b = \frac{612 \div 9}{999 \div 9} = \frac{68}{111}$ So $0.\dot{6}1\dot{2}$ converted into a fraction is $\frac{68}{111}$.

Convert these recurring decimals into fractions

1	$0.\dot{1}\dot{2}$	11	$0.\dot{1}\dot{7}$
2	$0.\dot{1}3\dot{2}$	12	$0.\dot{1}9128\dot{}$
3	$0.\dot{1}7\dot{1}$	13	$0.\dot{1}461\dot{3}$
4	$0.\dot{1}9\dot{8}$	14	$0.\dot{3}421\dot{8}$
5	$0.\dot{2}5\dot{2}$	15	$0.\dot{7}6\dot{9}$
6	$0.\dot{1}\dot{6}$	16	$0.\dot{8}35\dot{4}$
7	$0.\dot{7}654\dot{2}$	17	$0.\dot{3}652\dot{7}$
8	$0.\dot{1}984\dot{2}$	18	$0.\dot{5}83\dot{6}$
9	$0.\dot{6}132\dot{7}$	19	$0.\dot{7}264\dot{5}$
10	$0.\dot{2}2314\dot{9}$	20	$0.\dot{6}23\dot{1}$

Success Criteria for Converting Recurring Decimals to Fractions (the More Challenging Type)

Step	Instruction	Example
1	Label the whole decimal fraction. Here, I have chosen a as my label.	$a = 0.13\dot{7}4\dot{1}$
2	Split the fraction into the nonrecurring and the recurring parts	$a = 0.13 + 0.00\dot{7}4\dot{1}$
1	Label the recurring part of your fraction with a letter. Here, I have chosen b.	Convert $0.00\dot{7}4\dot{1}$ into a fraction. Let $b = 0.00\dot{7}4\dot{1}$
2	Count the number of digits in the string from the tenths column to the last recurring digit.	First digit is 0, second digit is 0, third digit is 7, the fourth digit is 4, the fifth digit is 1 so there are 5 digits in the string.
3	Multiply b by 10^n where n is the number that you found in step 2	$100000b = 741.\dot{7}4\dot{1}$
4	Subtract b from both sides to leave just an integer on the right	$1000b - b = 741.\dot{7}4\dot{1} - 0.\dot{7}4\dot{1}$ $99999b = 741$
5	Rearrange the equation to get b= a fraction	$b = \frac{741}{99999}$
6	Simplify the fraction by cancelling it down	$b = \frac{741 \div 3}{99999 \div 3} = \frac{247}{33333}$
7	Change the first part of the decimal that was not part of the recurring fraction to a fraction by counting the digits after the decimal point and putting 10^n where n is the number of digits that you counted.	$0.13 = \frac{13}{10^2} = \frac{13}{100}$
8	Add the two fractions together to give you a fraction for the value of a.	$\frac{13}{100} + \frac{247}{33333} = \frac{(13 \times 33333) + (100 \times 247)}{100 \times 33333}$ $= \frac{458029}{3333300}$ So $0.13\dot{7}4\dot{1}$ written as a fraction is $\frac{458029}{3333300}$.

Convert these recurring decimals into fractions

1	$0.2\dot{1}0\dot{2} =$	5	$0.16\dot{4}\dot{0} =$	9	$0.531\dot{7}6\dot{6} =$
2	$0.5\dot{3}1\dot{9} =$	6	$0.18\dot{0}\dot{3} =$	10	$0.72\dot{1}00\dot{7} =$
3	$0.8\dot{7}9\dot{7} =$	7	$0.35\dot{0}2\dot{7} =$	11	$0.188\dot{9}0\dot{3} =$
4	$0.0\dot{2}4\dot{0} =$	8	$0.418\dot{8}8\dot{4} =$	12	$0.934\dot{5}8\dot{5} =$

Simplifying Algebraic Fractions

Success Criteria to Simplify Algebraic Fractions (Easy Type)

Step	Instruction	Example
1	For each individual letter, use the laws of indices to cancel each one down.	$\frac{x^2y^3z^5}{x^5y^2z^5} = \frac{y}{x^3} = x^{-3}y$

Simplify these Algebraic Fractions (Easy Type)

1 $\frac{x^3y^2}{x^2} =$

2 $\frac{x^5y^7}{y^6} =$

3 $\frac{x^3y^5}{x^6} =$

4 $\frac{x^8y^3z}{x^8y^3} =$

5 $\frac{p^{12}q^5r^3s^4}{p^2qr^5} =$

6 $\frac{12a^3b^2}{8c^2} =$

7 $\frac{6x^3y^2}{4x^2} =$

8 $\frac{9x^3y^2}{15x^2y^4} =$

9 $\frac{(p^3q^2)^2}{p^8} =$

10 $\left(\frac{x^3y^2}{x^2}\right)^2 =$

Success Criteria to Simplify Algebraic Fractions (Harder Type)

Step	Instruction	Example
1	Factorise the numerator and denominator using the grid method	$\frac{x^2-3x-10}{x^2-4}$ $=\frac{(x+2)(x-5)}{(x+2)(x-2)}$
2	Cross out the common factor from the numerator and the denominator.	$\frac{\cancel{(x+2)}(x-5)}{\cancel{(x+2)}(x-2)}$
3	Write out the numerator and denominator.	$\frac{x^2-3x-10}{x^2-4}$ $=\frac{x-5}{x-2}$

Simplify Algebraic Fractions (Harder Type)

1 $\frac{10x^2-13x-3}{6x^2-x-12}=$

2 $\frac{18x^2+18x-8}{6x^2-13x-28}=$

3 $\frac{15x^2-17x-4}{6x^2-32x+32}=$

4 $\frac{21x^2-22x-8}{14x^2-45x-14}=$

5 $\frac{15x^2+x-6}{20x^2-22x+6}=$

6 $\frac{35x^2+13x-12}{35x^2-43x+6}=$

Challenge Question

C1 $\frac{60x^3+78x^2+12x-6}{40x^3-128x^2-46x-14}=$

Simplifying Surds by Rationalising the Denominators

Rationalising just means multiplying by 1. How you present the one is your choice but some variations will help you to simplify the surds and others won't.

The aim is to get rid of the surd in the numerator.

Step	Instruction	Example
1	Multiply the Surds by 1 in the form of $\frac{denominator}{denominator}$.	$\frac{2\sqrt{3}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$
2	Simplify the fraction.	$\frac{2\sqrt{3}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{15}}{5}$
	Slightly More Difficult Example	
1	Sometimes, you will be able to simplify the fraction first.	$\frac{\sqrt{60}}{\sqrt{35}} = \frac{\sqrt{12}}{\sqrt{7}} = \frac{2\sqrt{3}}{\sqrt{7}}$
2	Rationalise the denominator by multiplying by 1.	$\frac{2\sqrt{3}}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{2\sqrt{21}}{7}$

Simplifying Surds by Rationalising the Denominator

1 $\frac{\sqrt{5}}{\sqrt{3}} =$

2 $\frac{\sqrt{8}}{\sqrt{5}} =$

3 $\frac{\sqrt{15}}{\sqrt{7}} =$

4 $\frac{\sqrt{24}}{\sqrt{7}} =$

5 $\frac{\sqrt{8}}{\sqrt{11}} =$

6 $\frac{\sqrt{18}}{\sqrt{12}} =$

7 $\frac{\sqrt{50}}{\sqrt{3}} =$

8 $\frac{\sqrt{16}}{\sqrt{88}} =$

9 $\frac{\sqrt{72}}{\sqrt{128}} =$

10 $\frac{\sqrt{18}}{\sqrt{28}} =$

11 $\frac{\sqrt{24}}{\sqrt{48}} =$

12 $\frac{\sqrt{51}}{\sqrt{34}} =$

13 $\frac{\sqrt{60}}{\sqrt{80}} =$

14 $\frac{\sqrt{45}}{\sqrt{30}} =$

15 $\frac{\sqrt{360}}{\sqrt{240}} =$

Simplifying Surds by Rationalising the Denominator with Two Terms in the Denominator

The conjugate of an expression can be explained most clearly by example:

The **conjugate of $a + b$ is $a - b$**. You will notice the only difference here is that the sign has changed.

The conjugate of $3 + \sqrt{2}$ is $3 - \sqrt{2}$.

The conjugate of $4 - \frac{\sqrt{5}}{2}$ is $4 + \frac{\sqrt{5}}{2}$.

I'm hoping you have noticed that the only thing that changes is the sign in the middle.

Success Criteria for Rationalising Surds with Two Terms in the Denominator

Step	Instruction	Example
1	Multiply the fraction by one in the form of $\frac{Conjugate\ of\ the\ denominator}{Conjugate\ of\ the\ denominator}$	$\frac{7}{2-\sqrt{7}} \times \frac{2+\sqrt{7}}{2+\sqrt{7}}$
2	Use the grid method to multiply the numerators by each other.	$\begin{array}{c\|c\|c} \times & 2 & +\sqrt{7} \\ \hline 7 & 14 & +7\sqrt{7} \end{array}$ $14 + 7\sqrt{7}$
3	Use the grid method to multiply the denominators together.	$\begin{array}{\|c\|c\|c\|} \hline \times & 2 & +\sqrt{7} \\ \hline 2 & 4 & +2\sqrt{7} \\ \hline -\sqrt{7} & -2\sqrt{7} & -7 \\ \hline \end{array}$ $4 + 2\sqrt{7} - 2\sqrt{7} - 7 = -3$
4	Write your fraction out and see if you can cancel down at all. The aim is to get rid of the surds in the denominator.	$\frac{14 + 7\sqrt{7}}{-3}$

Rationalise these Surds

1 $\frac{2}{5-\sqrt{2}} =$

2 $\frac{8}{6+\sqrt{3}} =$

3 $\frac{11}{8-\sqrt{5}} =$

4 $\frac{10}{5-\sqrt{5}} =$

5 $\frac{2+\sqrt{2}}{5-\sqrt{2}} =$

6 $\frac{7-\sqrt{5}}{5(1+\sqrt{5})} =$

7 $\frac{3+2\sqrt{3}}{4-\sqrt{7}} =$

8 $\frac{9+\sqrt{27}}{3+\sqrt{3}} =$